Prof Vidya Sagar
Department of physics
B.Sc (part-1) Hons

Addition of velocities:-

Very high velocities cannot be added directly as in the classical mechanics, because this would lead to a violation of the postulates of relativity. Such velocities must be added in a manner consistent with the Lorentz transformation.

Let there be two frames of reference S and S', the frame S' is moving with a constant velocity v relative to S along to x-axis. Let a body moves a distance dx in time dt in the frame S. ~~along~~ Then the velocity of the body measured by observer in S is

$$u = \frac{dx}{dt}$$

To an observer in S', both the distance and the time interval will appear different say, dx' and dt'. So that to him the velocity of the body will be

$$u_1 = \frac{dx'}{dt'}$$

∴ From Lorentz transformation equations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \; dx' = \frac{dx - vdt}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t' = \frac{dt - vdx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \; u_1 = \frac{dx'}{dt'} = \frac{dx - vdt}{dt - vdx/c^2}$$

Dividing by dt we get

$$u_1 = \frac{\frac{dx}{dt} - v}{1 - (v/c^2)\frac{dx}{dt}}$$

But $\frac{dx}{dt} = u$

$$\therefore \; u_1 = \frac{u - v}{1 - uv/c^2}$$

This is the relativistic addition of velocities u and v.